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THE STUDY OF CURRENT-VOLTAGE CHARACTERISTICS OF TWO-DIMENSIONAL GRAPHENE-BASED SUPERLATTICE

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ИССЛЕДОВАНИЕ ВОЛЬТ-АМПЕРНОЙ ХАРАКТЕРИСТИКИ ДВУМЕРНОЙ СВЕРХРЕШЕТКИ НА ОСНОВЕ ГРАФЕНА

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Abstract. In this paper the process of current flow in the layer of graphene is investigated. The energy spectrum of graphene supposed to be modulated with a periodic potential of the substrate in two mutually perpendicular directions (one of the ways to create two-dimensional superlattices). The effect of the cross-current constant electric field is studied. It is shown that the presence of the perpendicular electric field leads to the appearance of an additional peak in the current-voltage characteristics of the superlattice.

Key words: current-voltage characteristic, two-dimensional superlattice, graphene, constant electric field.

Аннотация. В данной работе исследуется процесс протекания тока в слое графена, энергетический спектр которого модулирован периодическим потенциалом подложки в двух взаимно перпендикулярных направлениях (один из способов создания двумерной сверхрешетки). Рассмотрено влияние на ток поперечного постоянного электрического поля. Показано, что наличие поперечного поля приводит к появлению дополнительного максимума в вольт-амперной характеристике сверхрешетки.

Ключевые слова: вольт-амперная характеристика, двумерная сверхрешетка, графен, постоянное электрическое поле.

The last decade was marked by a surge of researchers interest to materials based on graphene – a monolayer of graphite [1-3]. Particularly, a number of ways were proposed to produce graphene-based superlattices. One of these variants consists in coating graphene on a substrate of alternating layers of silicon oxide and hexagonal boron nitride [4, 5].

In this article we consider a superlattice based on graphene placed on a substrate where SiO_2 and h-BN cells alternate in a checkerboard pattern, forming a periodic structure with a period of d_1 along one axis (the Ox axis, in this case) and the period of d_2 along the other (respectively, along the axis Oy). Let’s consider the effect of constant electric field $\vec{E}(E_x, E_y, 0)$ on the superlattice, provided that electric current can flow only in the direction of the Ox axis (see fig.1).

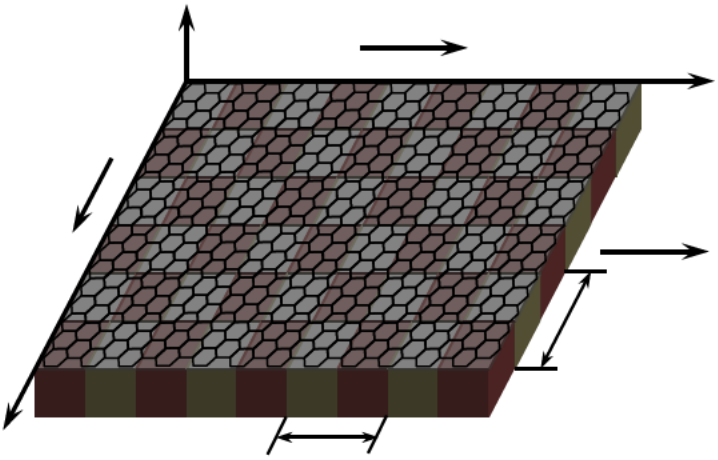


Figure.1. Geometry of the problem.

In this case, the current density is given by (we can put the value of $\hbar = 1$):

$$j_x = \frac{2e}{(2\pi)^3} \int_{\vec{p}} v_x(\vec{p}) f(\vec{p}, t) d\vec{p}, \qquad (1)$$

where the electron velocity is the momentum derivative of the energy and the distribution function $f(\vec{p}, t)$ is the solution of the Boltzmann kinetic equation

$$\frac{\partial f}{\partial t} + e\vec{E}\frac{\partial f}{\partial \vec{p}} = -\nu(f(\vec{p}) - f_0(\vec{p})). \qquad (2)$$

The right part of this equation was taken in approximation of the constant frequency of interactions ν. $f(\vec{p})$ and $f_0(\vec{p})$ are nonequilibrium and equilibrium distribution functions respectively. It is known that the solution of the Boltzmann kinetic equation could be written as:

$$f(\vec{p}, t) = \nu \int_{-\infty}^{t} dt' \exp(-\nu(t - t')) f_0(\vec{p}'(t', \vec{p}, t)), \qquad (3)$$

where $\vec{p}'$ is a solution of the classic equation of the electron motion

$$\frac{d\vec{p}'}{dt} = e\vec{E} \qquad (4)$$

with initial conditions $t' = t$ и $\vec{p}' = \vec{p}$, and the equilibrium distribution function is determined as:

$$f_0(\vec{p}) = C \cdot \exp\left(-\frac{\varepsilon(\vec{p})}{kT}\right), \qquad (5)$$

here C is a normalization constant.

Taking the energy spectrum of electrons in the form

$$\varepsilon(\vec{p}) = \sqrt{\Delta^2 + \Delta_1^2(1 - \cos(p_x d_1)) + \Delta_2^2(1 - \cos(p_y d_2))}, \qquad (6)$$

we obtain an expression for the velocity of the electron:

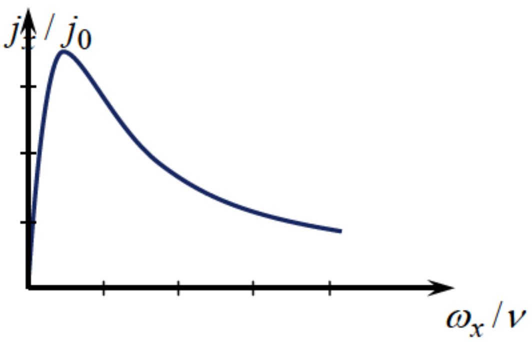
$$v_x(\vec{p}) = \frac{\partial \varepsilon(\vec{p})}{\partial p_x} = \frac{\Delta_1^2 d_1 \sin(p_x d_1)}{2\sqrt{\Delta^2 + \Delta_1^2(1-\cos(p_x d_1)) + \Delta_2^2(1-\cos(p_y d_2))}}. \qquad (7)$$

After determining the normalization constant in (5) we can substitute the obtained expressions (3) – (7) in the expression for the current density (1). Then resulting integrals to the dimensionless quantities we obtain:

$$j_x = j_0 \int_0^{\infty} dt \exp(-t) \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} d\alpha\, d\beta \frac{\sin\alpha \exp\left(-\frac{D}{kT}\sqrt{g(\alpha,\beta,\omega_x,\omega_y)}\right)}{\sqrt{1-\frac{\Delta_1^2}{D^2}\cos\alpha - \frac{\Delta_2^2}{D^2}\cos\beta}}, \qquad (8)$$

here $j_0 = \frac{e\Delta_1^2 d_1 n_0 \exp(D/kT)}{8\pi^2 D \cdot I_0(\Delta_1^2/(2DkT)) \cdot I_0(\Delta_2^2/(2DkT))}$, $D = \sqrt{\Delta^2 + \Delta_1^2 + \Delta_2^2}$, $\omega_x = eE_x d_1$, $\omega_y = eE_y d_2$, $g(\alpha,\beta,\omega_x,\omega_y) = 1 - \frac{\Delta_1^2}{D^2}\cos\left(\alpha + \frac{\omega_x}{\nu}t\right) - \frac{\Delta_2^2}{D^2}\cos\left(\beta + \frac{\omega_y}{\nu}t\right)$, $I_0(x)$ – the modified Bessel function.

The right numerical integration in accordance to expression (8) allows us to analyze the nature of changes in current density depending on the magnitude of the intensities of the electric field components in the same direction as current and perpendicular to it (see fig.2).



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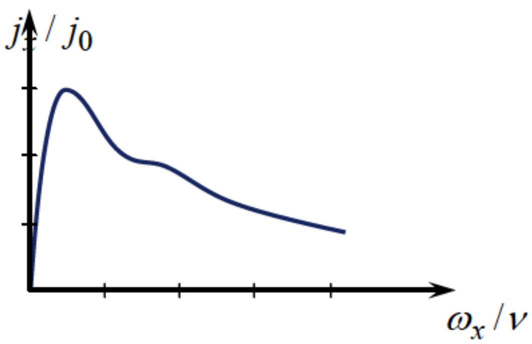

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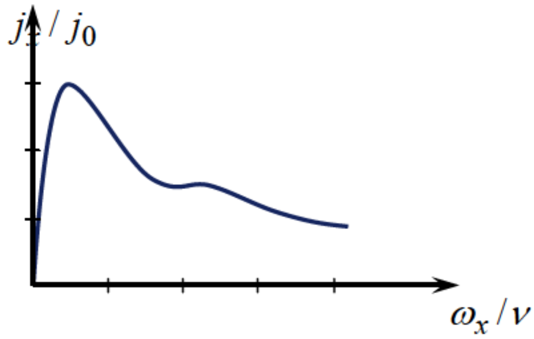

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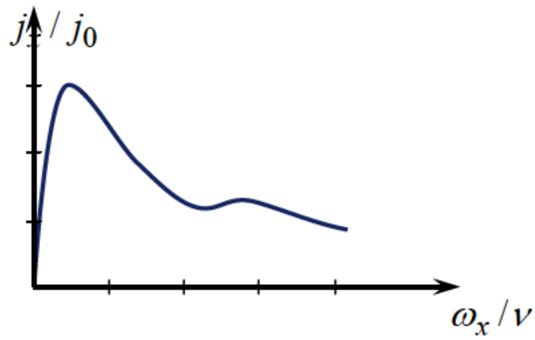

d)

Figure.2. Graphs of current density depending on the ratio ω_x / ν for different values of ω_y / ν: a) $\omega_y / \nu = 0$; b) $\omega_y / \nu = 3$; c) $\omega_y / \nu = 4$; d) $\omega_y / \nu = 5$.

Integrating (8) using the expansion of the root and the exponent functions in a row leads to the following expression for current density:

$$j_x = j_0 \left(C_1 \frac{\omega_x}{\omega_x^2 + \nu^2} + C_2 \left(\frac{\omega_x - \omega_y}{(\omega_x - \omega_y)^2 + \nu^2} + \frac{\omega_x + \omega_y}{(\omega_x + \omega_y)^2 + \nu^2} \right) \right), \qquad (9)$$

constants C_1 and C_2 depend just on the physical properties of the superlattice.

Using the expression (9) it is easier to predict the behavior of the system depending on the transverse electric field value (fig.3).

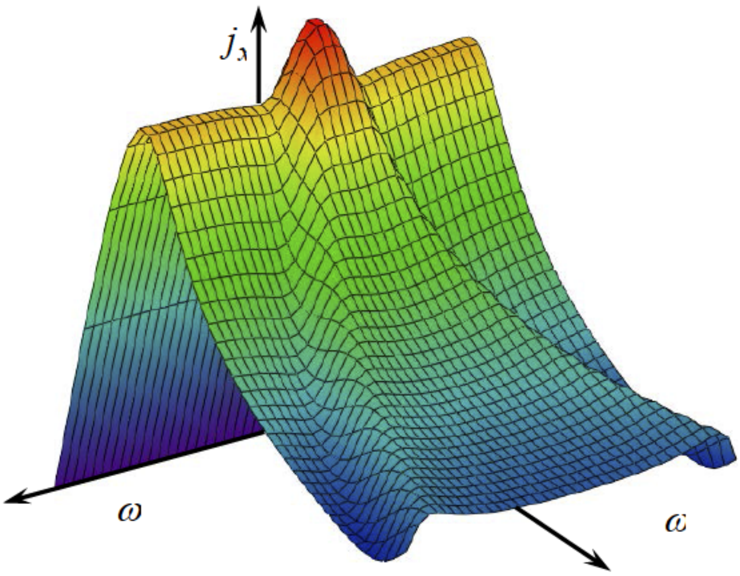

Figure.3. 3-d graph of current density constructed according to (9) with the range of values: ω_x / ν from 0 to 10 and ω_y / ν from -10 to 10.

Numerical analysis of (8) and a study of the expression (9) shows that in the presence of the transverse electric field component, an additional maximum appears on the current-voltage characteristic slightly shifted from the value of ω_y / ν in the direction of a strong field. In addition, it is easy to notice from the figure 3 that the appearance of even a small field along the Oy axis leads to a decrease in the main peak current density , which is obviously due to the decrease in the number of charge carriers.

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ONE CASE OF ASYMPTOTIC BEHAVIOR OF THE PROBABILITY OF BANKRUPTCY OF INSURANCE COMPANY

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ОДИН СЛУЧАЙ АСИМПТОТИКИ ВЕРОЯТНОСТИ БАНКРОТСТВА СТРАХОВОЙ КОМПАНИИ

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Abstract. In this paper we find the asymptotic of the probability of bankruptcy for large payments when payments have Weibull distribution and Benktander type I.

Key words: probability of bankruptcy, "heavy tails", subexponential distribution.

Аннотация. В данной работе мы находим асимптотику вероятности банкротства для крупных платежей, когда платежи имеют распределение Вейбулла и Буктандера типа I.

Ключевые слова: вероятность банкротства, "тяжелые хвосты", субэкспоненциальное распределение.

Activity of insurance company characterized by different parameters, one of which is the probability of bankruptcy. We can say that the financial risk and the related risk of bankruptcy – the characteristics of each insurance company. Hence, an important task is to calculate the probability of bankruptcy and analysis of the results. Special interest, in our time, is determining the probability of bankruptcy for large payments, connected to natural disasters, terrorist acts, etc. In this article, we will look at this particular case when large payments. Note also that large payment distributions described the so-called "heavy tails".

In the analysis of such distributions, including the distribution of Pareto, the question arose whether it is possible to obtain an estimate of probability of bankruptcy $\varphi(u)$. A positive response was given to this question von Bahr [1] for the Pareto distribution and Thorin and Vikstad [2] for the log-normal distribution.

Later there was a question whether there is a class of distributions with "heavy tails", which allows finding the probability of bankruptcy. The answer is given by Embrehts and Vereverbeke [3], which revealed a fundamental role of class subexponential distributions S in the theory of risk. This class include the log-normal distribution, distribution Pareto distribution Barra, log gamma distribution, cut resistant distribution, Weibull distribution, distribution Bektandera type I and type II.

Suppose that we are in the classic problem of finding the probability of bankruptcy (see. In particular [4] p. 184-186).

The classic model of collective risk is characterized by:

1. Payment size $\{X_i, i \geq 1\}$ is non negative independent identically distributed random variables with distribution function $F(x)$ and finite expectation $\mu = EX_1$.

2. The time of receiving payment claims $\{T_i, i \geq 1\}$ for forming a sequence of independent identically distributed random variables with distribution function $F(x)$.

3. The process flow requirements for payments $N(t) = \sup\{n \geq 1 : T_n \leq t\}$, $t > 0$, namely, the number of requirements on the range $[0, t]$, where, by definition, sup{∅}=0.

4. The time between arrival of the requirements $Y_1 = T_1$, $Y_k = T_k - T_{k-1}$, $k \geq 2$ - independent identically distributed random variables with finite expectation $EY_1 = 1/\lambda$.

5. $u \geq 0 -$ initial (reserve) capital.

6. $c > 0$ - speed (intensity) of insurance premiums.

Let

1) $\varphi(u,T)$ = P {$U(t)$<0 for some $0 < t \leq T$ }, $0 < t < \infty$, $u > 0 -$ probability of bankruptcy on a finite time interval $[0, T]$, $U(t)$ - the process of risk;

2) $\varphi(u) = \varphi(u, \infty) =$ P {$U(t)$<0 for some $t > 0$ } – probability of bankruptcy on an infinite interval.

To calculate the probability of bankruptcy it is comfortable to have a simple analytical formulas for $\varphi(u)$ or $\varphi(u,T)$ that include probabilistic characteristics of the insurance payments and process flow requirements for payment $N(t)$.

Will use the following terms and symbols: if $F(x)$ - distribution function, $\overline{F}(x) = 1 - F(x)$ the "tail" of the distribution F, and a F^{n*} - n-fold convolution F.

So if F - distribution function of benefits, then $\overline{F}(x)$ - "tail" of the distribution, and

$$F_I(x) = \frac{1}{\mu}\int_0^x \overline{F}(y)dy, \; x > 0,$$

called integrated "tail" of the distribution. [4 p.186]

The value of $\rho = \dfrac{c}{\lambda\mu} - 1$ called the relative insurance premium and for the basic conditions $\rho > 0$ we use the term "a net profit".

Cramer-Lundberg’s condition implies the existence of constants ν called debug (regulator, adjusting) ratio or Lundberg’s ratio, such that

$$\int_0^\infty e^{\nu x}(1 - F(x))dx = c/\lambda = (1+\rho)\mu, \tag{1}$$

The distributions that do not satisfy the condition (1) will be called distributions with "heavy tails" [4 p.188]. These distributions, as mentioned above, are called subexponential distributions.

For further practical implementation of calculation of bankruptcy probability we use the following theorem (see in particular [4, p. 197]).

Theorem. Consider Cramer-Lundberg model under the conditions $\rho > 0$ and $F_I(\mathrm{x}) \in S$. Then

$$\varphi(u) \sim \rho^{-1}\overline{F_I}(u), \; u \to \infty \tag{2}$$

According to this theorem, in the case of payments which have distributions of subexponential integrated "tails", the probability of bankruptcy allows a simple approximation, given by the formula (2).

Note that the condition of the theorem formulated in terms integrated "tails" instead of the distribution function.

Calculating the probability of bankruptcy for large payments

Consider the problem of calculating the probability of bankruptcy in the case of "heavy tails", that is, when payments are large. Note that in [5], the case of Pareto distribution with distribution function $F(x)=1-\left(\frac{k}{k+x}\right)^{\alpha}, \alpha>1, k>0, x>0$ and Benktander distribution type II.

Statements 1. Let payments distributed by Weibull distribution with a parameter $0<\gamma<1$, and the distribution function

$$F(x)=1-\exp\left(-c_1 x^{\gamma}\right), c_1>0, x>0$$

then the asymptotics of probability of bankruptcy is given

$$\varphi(u)\sim\frac{\lambda}{\mathrm{c}\cdot\mathrm{c}_1^{\frac{1}{\gamma}}-\lambda\Gamma\left(1+\frac{1}{\lambda}\right)}\left[1+\frac{\Gamma\left(\frac{1}{\gamma};c_1x^{\gamma}\right)-\Gamma\left(\frac{1}{\gamma};0\right)}{\gamma\cdot\Gamma\left(1+\frac{1}{\gamma}\right)}\right],\ u\to\infty.$$

Statement 2. Let payments distributed Benktander type I:

$$1-F(x)=\left(1+\frac{2\beta\ln x}{\alpha}\right)x^{-(\alpha+1+\beta\ln x)}\ \alpha,\beta>0, x>1,$$

Asymptotics of probability of bankruptcy $\varphi(u)$ given by the following equation:

$$\varphi(u)\sim\frac{\lambda(\alpha+1\text{-}\mathrm{u}^{-\alpha-\beta\ln x})}{c\alpha-\lambda(\alpha+1)}, u\to\infty.$$

Proof: The function of distribution and density are as follows:

$$F(x)=1-\left(1+\frac{2\beta\ln x}{\alpha}\right)x^{-(\alpha+1+\beta\ln x)}\ \alpha,\beta>0, x>1,$$

$$f(x)=\left(\left[\left(1+\frac{2\beta\ln x}{\alpha}\right)(1+\alpha+2\beta\ln x)\right]-\frac{2\beta}{\alpha}\right)x^{-(2+\alpha+\beta\ln x)}.$$

Find the mathematical expectation: $\mu=EX=\int_1^{+\infty}xf(x)dx=\frac{\alpha+1}{\alpha}$

We find $\rho^{-1}=\frac{\lambda(\alpha+1)}{c\alpha-\lambda(\alpha+1)}$.

Find the integrated "tail" of the distribution:

$$F_I(x)=\frac{1}{\mu}\int_0^x\overline{F}(y)dy=\frac{x^{-\alpha-\beta\ln x}}{\alpha+1}$$

Accordingly, the asymptotic of probability of bankruptcy is given by the following equation:

$$\varphi(u) \sim \rho^{-1}\overline{F_I}(u) \sim \frac{\lambda(\alpha + 1 - \mathrm{u}^{-\alpha - \beta \ln x})}{c\alpha - \lambda(\alpha + 1)}, u \to \infty.$$

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